where $b = b_1$ or b_2 for the $\langle 100 \rangle$ problem or the $\langle 111 \rangle$ problem, respectively.

Consider the domain configuration in Fig. 2(b). Again the energy in the domain is

$$E_{me}(\text{domain}) = be \sin^2\theta .$$

The transition through the wall proceeds in the (xz) plane by varying continuously from $-\theta$ to θ. The energy in the wall is

$$E_{me}(\text{wall}) = be \sin^2\xi, \quad -\theta \le \xi \le \theta. \tag{20}$$

Equations (18)—(20) are the primary equations derived in this section.

B. Exchange Energy

Within the concepts of ferromagnetic domain theory, the exchange energy is believed to reside only in the domain walls or transition regions between adjacent domains. The method for obtaining this domain-wall energy is through a Landau—Lifshitz domain-wall calculation.[18] This has been fully developed in the literature[10,15] and will be described only briefly here. The method consists of writing a one-dimensional integral expression for the energy in the transition region between domains. The terms which contribute to the domain-wall energy are the exchange energy [Eq. (4)] and the excess crystalline or magnetoelastic anisotropy energy incurred by the transition through the wall. It is assumed that $\nabla \cdot \mathbf{M} = 0$ ($\theta = \text{const}$) holds through the wall. This one-dimensional integral energy expression is minimized by variational calculus. The result predicts that for all points within the wall the exchange energy is equal to the excess anisotropy energy. It is found that the domain-wall energy per unit area is given by[15]

$$\sigma_w = 2\sqrt{A} \sin\theta \int_{\Phi_1}^{\Phi_2} \left| [E_{me}(\text{domain}) - E_{me}(\text{wall})]^{1/2} \right| d\Phi . \tag{21}$$

The crystal anisotropy energy has not been considered. A is again the exchange constant and Φ_1 and Φ_2 are the azimuthal orientation of the magnetization in the adjacent domains separated by the wall.

Energies for domain walls oriented as shown in Figs. 2(a) and 2(b) will be called σ_w^s and σ_w^p, respectively. For Fig. 2(a), using Eqs. (18) and (19) with Eq. (21), gives

$$\sigma_w^s = 2(A|be|)^{1/2} \sin^2\theta \int_0^{\pi} \sin\Phi \, d\Phi$$

or

$$\sigma_w^s = 4(A|be|)^{1/2} \sin^2\theta . \tag{22}$$

For Fig. 2(b), using Eqs. (18) and (20) with Eq. (21), gives

$$\sigma_w^p = 2(A|be|)^{1/2} \int_{-\theta}^{\theta} (\sin^2\theta - \sin^2\xi)^{1/2} \, d\xi .$$

Making the substitution

$$\sin\xi = \sin\theta \sin x = a \sin x$$

and using the identity

$$\cos^2 x = (1 - a^{-2}) + a^{-2}(1 - a^2 \sin^2 x),$$

one obtains

$$\sigma_w^p = 4(A|be|)^{1/2} \left((a^2 - 1) \int_0^{\pi/2} \frac{dx}{(1 - a^2 \sin^2 x)^{1/2}} + \int_0^{\pi/2} (1 - a^2 \sin^2 x)^{1/2} \right) dx .$$

This is

$$\sigma_w^p = 4(A|be|)^{1/2} [(a^2 - 1)K(a, \tfrac{1}{2}\pi) + E(a, \tfrac{1}{2}\pi)], \tag{23}$$

where K and E are complete elliptic integrals of the first and second kind and $a = \sin\theta$.

σ_w^s and σ_w^p are compared in Fig. 3. It is seen that the domain model considered in Fig. 2(b) yields a slightly lower energy. In actual crystalline material imperfections such as dislocation, impurities, etc., can significantly alter the domain-wall energy. For this reason, it is believed that the slight energy difference is not substantial enough to favor the domain structure in Fig. 2(b) over that in Fig. 2(a). Alternatively, a Boltzmann distribution predicts, at normal temperatures, roughly an even distribution of domain walls of both orientations. From this, one may conclude that ferromagnetic domain theory suggests a needle- or sliver-shaped domain structure oriented along the axis of uniaxial strain will nucleate behind the shock front. A model for this structure is shown in Fig. 2(c).

Due to the much simpler form of Eq. (22), the approximation

$$\sigma_w^p \simeq \sigma_w^s = \sigma_w = 4(A|be|)^{1/2} \sin^2\theta$$

will be made. An expression for the effective exchange energy density in Eq. (1) can be obtained by dividing

$$\sigma_w^p + \sigma_w^s \simeq 2\sigma_w$$

by the domain dimension D shown in Fig. 2(c). This gives

$$E_{ex} = 2\sigma_w / D$$

or

$$E_{ex} = [8(A|be|)^{1/2}/D] \sin^2\theta . \tag{24}$$

C. Demagnetizing Energy

The demagnetizing energy can be obtained by solving the magnetostatic boundary value problem for the magnetic surface pole distribution on two surfaces separated a distance L as indicated in Fig. 4. The solution requires only a slight variation on a problem already solved by Kittel.[19] The result is

$$E_d = 1.1(DM_s^2/L) \sin^2\theta . \tag{25}$$

L is the slab thickness, D is the domain dimension, and M_s is the saturation magnetization.

D. Total Energy

From the results of this section, Eq. (1) for the total thermodynamic energy can now be explicitly written

$$E(D, \theta) = -M_s H_e \cos\theta + be \sin^2\theta + 1.1(DM_s^2/L) \sin^2\theta + [8(A|be|)^{1/2}/D] \sin^2\theta , \tag{26}$$

where $\cos\theta$ is the component of the magnetization in the direction of the applied field.

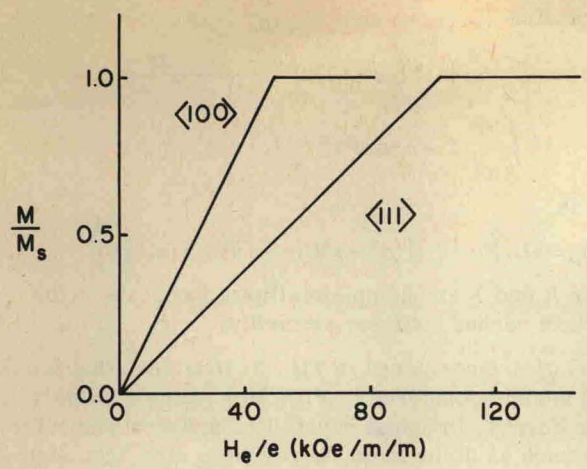

FIG. 6. Magnetization curves for the ⟨100⟩ and the ⟨111⟩ problem in YIG.

IV. EQUILIBRIUM MAGNETIC BEHAVIOR

Equilibrium thermodynamics requires that the energy expression $E(D,\theta)$ be a minimum with respect to a variation of the internal coordinates D and θ. Consider the domain-width parameter first. Minimizing with respect to D gives

$$\frac{\partial E}{\partial D}=1.1\frac{M_s^2}{L}\sin^2\theta-\frac{8(A|be|)^{1/2}}{D^2}\sin^2\theta=0.$$

This yields an expression for the domain width:

$$D=[8L(A|be|)^{1/2}/1.1M_s^2]^{1/2}. \tag{27}$$

This can be substituted back into Eq. (26) giving

$$E(\theta)=-M_sH_e\cos\theta+be\sin^2\theta+2[8.8M_s^2(A|be|)^{1/2}/L]^{1/2}\sin^2\theta \tag{28}$$

or

$$E(\theta)=-M_sH_e\cos\theta+be\sin^2\theta+\gamma|e|^{1/4}\sin^2\theta,$$

where

$$\gamma=2[8.8M_s^2(A|b|)^{1/2}/L]^{1/2}.$$

The last term in Eq. (28) will be called the equilibrium exchange and demagnetizing energy. Note that it increases as the fourth root of the strain.

From Eq. (28) the magnetization curve can be obtained. The requirement of equilibrium yields two solutions:

$$\sin\theta=0 \tag{29}$$

or

$$2(be+\gamma|e|^{1/4})\cos\theta+H_eM_s=0. \tag{30}$$

The correct solution for a given applied field is determined from the requirement that the equilibrium point be a minimum. Two cases occur which are determined by the sign of $\beta\equiv(be+\gamma|e|^{1/4})$. The sign depends on the strain e (usually negative in shock-wave experiments) and the magnetoelastic constant b. γ is always positive. For the first case ($\beta<0$), the magnetization is

$$\begin{aligned} M/M_s&=1, && H_e>-2\beta/M_s\\ &=-(M_s/2\beta)H_e, && H_e<-2\beta/M_s. \end{aligned} \tag{31}$$

The form of the magnetization curve is shown in Fig. 5. Shock-induced demagnetization is expected for $\beta<0$. For example, referring to Fig. 5, a material initially in magnetic saturation in a transverse field H_{e0} would suffer a reduction in magnetization to a value M after passage of the shock wave. For the second case ($\beta>0$), the axis of uniaxial strain defines a hard direction of magnetization. All perpendicular axes are equivalent easy directions and the magnetization curve predicted by this simple treatment will be a discontinuous jump of $2M_s$ on reversal of the applied field. This case does not, however, lead to shock demagnetization and, therefore, is not of interest in the present context.

V. DISCUSSION

Results of Sec. IV will be considered using the material properties of yttrium iron garnet. This ferrimagnetic ceramic has received attention in previous shock-induced anisotropy work because of the convenient magnitudes of its material properties. A rough value for the exchange constant of YIG obtained from molecular field theory is $A\approx3\times10^{-7}$ erg/cm. At a strain of -0.01 in YIG, which corresponds to about 25-kbar shock pressure, the predicted domain width from Eq. (27) is 20 μ. This is in agreement with other work.[7]

The equilibrium exchange and demagnetizing energy [last term in Eq. (28)] is observed to increase as the fourth root of the strain while the induced anisotropy energy increases linearly with the strain. This implies that the equilibrium exchange and demagnetizing energy would assume decreasing importance with increasing strain. For a strain of -0.01, the equilibrium exchange and demagnetizing energy is about 2% of the induced anisotropy energy. This justifies the approximation of ignoring this energy term in predicting magnetic behavior in the region of large strain as has been done in previous work.[2,5,7] It is worth noting that this approximation does not extend to all materials. In iron this neglected term represents a significant part of the energy even up to the elastic limit of the material.

Magnetization curves for the ⟨100⟩ problem and the ⟨111⟩ in YIG are shown in Fig. 6. They will be referred to in the following article.[11] The equilibrium exchange and demagnetizing energy has been ignored. The curves are plotted against the parameter H_e/e. Magnetization curves for any strain are expected to be self-similar against this parameter.

VI. SUMMARY

(i) The fourth-rank magnetoelastic tensor for a given state of uniaxial strain can be analyzed with the familiar techniques available for second-rank symmetric tensors. It was found that the axis of uniaxial strain defined an easy or hard direction of magnetization only in special cases of particular crystal orientations such as uniaxial strain along the ⟨100⟩ or ⟨111⟩ directions or in the case of magnetoelastic isotropy.

(ii) The established methods of ferromagnetic domain